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The application of the rate theory of deformation to the yield behaviour of wood

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The yield behaviour of wood is assumed to result from the stress-assisted thermally activated motion of elementary fibrils over short-range barriers. The barrier energy is taken to be a linear decreasing function of increasing moisture content and the activation volume a linear increasing function of increasing moisture content. The flow stress equation obtained from the application of the rate theory is given by

$$\tau(T,W) = \tau_a + \frac{H(T,0)}{v(0)} \frac{\left(1 - a_T \dfrac{W}{W_f}\right)}{\left(1 + s_v \dfrac{W}{W_f}\right)} + \frac{KT \ln \dot{\gamma}/\dot{\gamma}_0}{v(0)\left(1 + s_v \dfrac{W}{W_f}\right)}.$$

The theory has been examined in relation to the strain-rate and moisture dependence of the maximum longitudinal crushing stress of *Pinus radiata* and *Kahikatea*. The data for these woods follow fairly closely the general expectations of the theory and the measured activation volumes are not inconsistent with the thermally activated motion of elementary fibrils.

1. Introduction

The structure of wood is complicated and its mechanical behaviour sensitive to the variables of time, temperature and moisture content. In this paper we examine the yield behaviour of soft woods as a function of strain-rate, temperature and moisture content in terms of the rate theory of deformation. Before proceeding with the application of the rate theory we will briefly examine the typical structure of softwoods. The basic structure of a softwood [1] consists of longitudinal or tracheid cells that grow vertically up the tree. There are other cell types but the tracheid cells usually make up over 90% of the total wood volume. In cross-section the tracheid is known to be made up of an inner secondary wall and an outer primary wall. Surrounding the primary wall is a layer called the middle lamella which joins the cells together. The bulk of the cell wall consists of the secondary layer. The cell walls are composed of three groups of structural substances which have been classed as framework, matrix and encrusting materials. The framework substance is cellulose which occurs in the form of microfibrils. The matrix substances which are incorporated in the cell walls are mainly hemicelluloses and the encrusting materials, lignins. The cellulose contributes to the strength and the lignin provides rigidity to the structure. In softwoods about 42% of the structure consists of cellulose and between 25 to 35% lignin of which 90% is concentrated in the middle lamella and primary wall. Hence to understand the deformation of wood we need only consider the structure as being composed of cellulose microfibrils embedded in some matrix material [2].

A further understanding of the deformation process can be obtained from an examination of the microfibril structure. Microfibrils are composed of aggregates of elementary fibrils and have diameters ranging from 100 to 300 Å. The microfibrils further aggregate to form what are sometimes called macrofibrils and these are formed into lamellae that make up the cell wall layers. The elementary fibrils are thought to be the cellulosic strands of the smallest possible diameter. They are thought to be about 30 Å diameter and to contain about 40 cellulose chains [3]. Recent work by Mark [4] and Fengel [5] would indicate that the elementary fibril diameter could be as small as 10 to 20 Å. The elementary fibrils contain regions of highly crystalline material and regions of amorphous or paracrystalline material. There is considerable evidence to suggest that a paracrystalline phase exists between elementary fibrils and within the microfibril. In these regions the cellulose chains are less highly ordered than within the crystalline portion of the elementary fibril and it is thought that they may be in close association with matrix substances and lignin.

Peterlin [6] has concluded that the creep and failure of strained fibrous material results from the sliding motion of the microfibrils and fibrils (the smallest structural unit in this case being taken as the microfibril and aggregates of these as fibrils) and that the molecules that tie the microfibrils together, tie molecules, have little effect on the behaviour. The resistance to deformation is assumed to arise from quasi-viscous type frictional forces which act on the extremely large surfaces of the long narrow microfibrils and fibrils. The frictional forces are assumed to result mainly from Van der Waal type adhesion. For wood we will assume that deformation results from the sliding motion of, in the main, the elementary fibrils and that the frictional forces acting on the surface of these fibrils result mainly from Van der Waal type forces.

Wood is hydrophilic and readily absorbs water up to its fibre saturation point. The water is absorbed in the amorphous region of the microfibrils and on the surface of these fibrils [7]. It is not absorbed in the crystalline regions. This absorption of water leads to swelling of the wood and conversely, desorption leads to shrinkage. The absorbed water acts as a plasticizer in that the strength properties decrease significantly as the moisture content is increased up to the fibre saturation point. This occurs because as the wood swells the distance between the microfibrils increases thus reducing the intermolecular forces such as the Van der Waal forces.

2. Theory

In the application of the rate theory to the deformation of wood we assume, as mentioned above, that the deformation results from the sliding motion of, in the main, the elementary fibrils and that the resistance to deformation arises from frictional forces acting on the outer surface of these fibrils. We further assume that the frictional forces can be considered as short-range barriers that are surmounted by stress assisted thermal fluctuations, thus giving rise to thermally activated fibril motion. We also assume the deformation is controlled by a single activation process having a barrier energy $H(0)$. The energy $H(0)$ should not be a function of stress but is expected to vary with temperature and moisture content, decreasing with increasing temperature and moisture content. This decrease just reflects the decrease in bond strength with either increasing temperature or increasing moisture content. In the deformation of metals it is usual to take the temperature variation of the barrier energy, $H(0)$, to be the same as that for the shear modulus, G, as the modulus reflects the bond strength [8]. Thus we have

$$H(T) = H(0)\,\frac{G(T)}{G(0)} \qquad (1)$$

where $H(T)$ is the barrier energy at T K, $H(0)$ is the barrier energy at 0 K, $G(T)$ is the shear modulus at T K, and $G(0)$ is the shear modulus at 0 K. For woods there will also be a variation of $H(0)$ with moisture content, thus

$$H(T,W) = H(0,0) \cdot \frac{G(T,W)}{G(0,0)} \qquad (2)$$

where W is the moisture content as determined on a dry weight basis. The modulus of wood varies in an almost linear manner with moisture content [1] up to the fibre saturation point and thereafter remains essentially independent of moisture. If to a first approximation we assume this variation to be linear then we can obtain a simple expression for $H(T,W)$ as a function of moisture content. From Fig. 1 we can see that

$$G(T,W) = -\frac{G(T,0) - G(T,W_f)}{W_f}\,W + G(T,0)$$

$$\therefore \frac{G(T,W)}{G(T,0)} = 1 - \frac{G(T,0) - G(T,W_f)}{G(T,0)}\,\frac{W}{W_f}$$

$$\therefore \frac{G(T,W)}{G(T,0)} = 1 - a_T\,\frac{W}{W_f} \qquad (3)$$

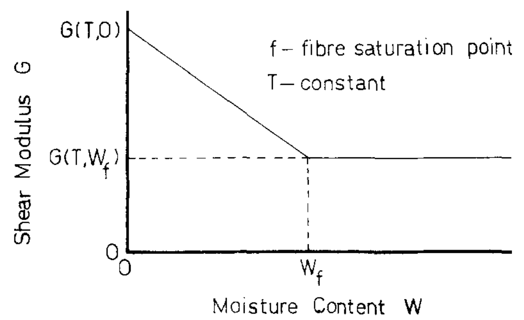

Figure 1 Variation of shear modulus with moisture content.

where $a_T^* = [G(T,0) - G(T,W_f)]/G(T,0)$ and may be a function of temperature. Hence it follows that

$$H(T,W) = H(0,0)\frac{G(T,0)}{G(0,0)}\left(1 - a_T\frac{W}{W_f}\right) \quad (4)$$

or

$$H(T, W) = H(T,0)\left(1 - a_T\frac{W}{W_f}\right) \quad (5)$$

where $H(T,0) = H(0,0)G(T,0)/G(0,0)$ and expresses the temperature variation of barrier energy.

A diagramatic representation of the barrier energy as a function of moisture content is shown in Fig. 2.

In the application of the rate theory to the deformation of metals the assumption is usually made that the applied stress, τ, may be regarded as the sum of an "athermal" stress, τ_a, and a local stress, τ^* [9], i.e.

$$\tau^* = \tau - \tau_a \quad (6)$$

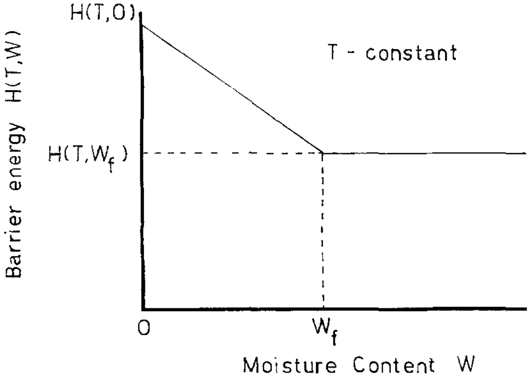

Figure 2 Short-range barrier energy as a function of moisture content.

where τ_a is a measure of the long-range internal stresses in the material and τ^* is the effective stress which, with the aid of thermal fluctuations, enables dislocations to surmount short-range barriers. We will assume the same behaviour for wood only τ^* will be taken as the effective stress assisting in the thermally activated motion of the elementary fibrils.

From the rate theory of deformation we obtain

$$\dot{\gamma} = \dot{\gamma}_0 \exp[-\Delta G(\tau^*, T, W)/KT] \quad (7)$$

where $\dot{\gamma}$ is the shear strain-rate, $\dot{\gamma}_0$ the pre-exponential or structure factor, which is taken as being proportional to the density of activation sites and the distance moved by an elementary fibril after a successful activation, and $\Delta G(\tau^*, T, W)$ is the free energy of activation. Since $\dot{\gamma}_0$ is related to the material structure it may be governed by the deformation history but is usually assumed to be independent of stress, temperature and strain-rate. We will also assume for the present that it is independent of moisture content.

The activation enthalpy, $\Delta H(\tau^*, T, W)$, can be derived from Equation 7 for

$$\Delta G(\tau^*, TW) = \Delta H(\tau^*, T, W) - T\Delta S(\tau^*, T, W) \quad (8)$$

where $\Delta S(\tau^*, T, W)$ is the activation entropy and is defined as

$$\Delta S(\tau^*, T, W) = -\left[\frac{\partial G(\tau^*, T, W)}{\partial T}\right]_{\tau^*, W}. \quad (9)$$

Hence from Equations 7 to 9

$$\begin{aligned}\Delta H(\tau^*, T, W) &= \Delta G(\tau^*, T, W) + T\left[K \ln \dot{\gamma}/\dot{\gamma}_0 + KT\frac{\partial \ln\dot{\gamma}/\dot{\gamma}_0}{\partial T}\bigg|_{\tau^*, W}\right]\\ &= \Delta G(\tau^*, T, W) - \Delta G(\tau^*, T, W) + KT^2\frac{\partial \ln\dot{\gamma}/\dot{\gamma}_0}{\partial T}\bigg|_{\tau^*, W}\end{aligned}$$

$$\therefore \Delta H(\tau^*, T, W) = KT^2\frac{\partial \ln\dot{\gamma}/\dot{\gamma}_0}{\partial T}\bigg|_{\tau^*, W} \quad (10)$$

which is usually written as

$$\Delta H(\tau^*, T, W) = -\frac{\partial \ln\dot{\gamma}/\dot{\gamma}_0}{\partial(1/KT)}\bigg|_{\tau^*, W}. \quad (11)$$

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*Note: a_T will always be less than one.

From Equations 7 and 9 it can be shown that

$$\left.\frac{\partial \ln \dot{\gamma}/\dot{\gamma}_0}{\partial T}\right|_{\tau^*, W} = \frac{1}{KT}\,[K \ln \dot{\gamma}_0/\dot{\gamma} + \Delta S(\tau^*, T, W)]. \tag{12}$$

In the deformation of metals ΔS is assumed to be constant and independent of τ^* and T. It is also taken to be small compared with $K\ln \dot{\gamma}_0/\dot{\gamma}$ so that from Equation 12 it follows that

$$\Delta H(\tau^*, T, W) \simeq \Delta G(\tau^*, T, W). \tag{13}$$

For wood because of the long-chain nature of the structural elements ΔS may be neither constant nor small compared with $K\ln \dot{\gamma}_0/\dot{\gamma}$ but for the present we will assume Equation 13 holds approximately true.

The activation volume, v, is defined as

$$v = -\left.\frac{\partial \Delta G(\tau^*, T, W)}{\partial \tau^*}\right|_{T,W}. \tag{14}$$

Hence from Equation 7 we obtain

$$v = KT\left.\frac{\partial \ln \dot{\gamma}}{\partial \tau^*}\right|_{T,W} - KT\left.\frac{\partial \ln \dot{\gamma}_0}{\partial \tau^*}\right|_{T,W} \tag{15}$$

$$\simeq 2KT\left.\frac{\partial \ln \dot{\gamma}/\dot{\gamma}_0}{\partial \sigma^*}\right|_{T,W} \tag{16}$$

where $KT(\partial \ln \dot{\gamma}/\partial T)|_{T,W}$ is defined as the apparent activation volume, v_a. If $\dot{\gamma}_0$ is a constant, independent of τ^*, T and W then v_a equals v. The activation volume can also be defined as the work done by the stress field during the activation [10]. Let the stress acting on an element dl of the elementary fibril be τ_l, and let this element move a distance dx during the overcoming of the short-range barrier. Then the work done by the stress, $U(\tau^*, T, W)$ is given by

$$\begin{aligned} U(\tau^*, T, W) &= D\int\!\!\int \tau_l \, dl dx \\ &= D^* l^* x^* \tau^* \\ \therefore U(\tau^*, T, W) &= v^* \tau^* \end{aligned} \tag{17}$$

where D^* is the diameter of the fibril, l^* the total length of fibril involved in the activation, τ^* the mean effective stress acting over this length, and x^* is called the activation distance; the asterisks emphasize that the various parameters may be functions of stress, temperature and moisture content. If the shape of the short-range barrier (or force–displacement curve) is not a function of stress, then v^* is identical with v and is given by Equation 15. This is an assumption we make in the derivation of the barrier energy, $H(T, W)$. Since the stress field does work $\tau^* v^*$, during the activation process, the thermal free energy needed for activation is

$$\Delta G(\tau^*, T, W) = G(\tau^*, T, W) - U(\tau^*, T, W)$$

where $G(\tau^*, T, W)$ is the total energy required to overcome the short-range barrier. From Equations 13 and 17 and the assumptions made previously about the barrier energy, it follows that

$$\Delta H(\tau^*, T, W) = H(T, W) - v^* \tau^* \tag{18}$$

or

$$\Delta H(\tau^*, T, W) = H(T,0)\left(1 - a_T \frac{W}{W_f}\right) - v(T, W)\tau^*$$

The allowed variation of activation volume with temperature, other than that which results from a concealed stress dependence [10], is at variance with the earlier assumption that ΔS be constant and small compared to $K\ln \dot{\gamma}_0/\dot{\gamma}$. This is because Briggs and Campbell [11] have shown that an increase in activation volume with increasing temperature is equivalent to an increase in activation entropy with increasing stress. For woods the activation volume may well vary with temperature but until this can be substantiated experimentally we will assume, v is not a function of temperature and that the analysis leading to Equation 13 holds true. Thus for the assumptions made Equation 18 becomes

$$\Delta H(\tau^*, T, W) = H(T,0)\left(1 - a_T \frac{W}{W_f}\right) - v(W)\tau^*. \tag{19}$$

The assumed dependence of activation volume on moisture content is not unreasonable, because wood swells on the absorption of water. The absorption of water leads to an increase in diameter of the fibrils, especially in the amorphous regions and an increase in the interfibrillar spacing. This latter effect leads to a reduction in the intermolecular forces and consequently $H(T,0)$. A change in $H(T,0)$, with moisture content is likely to lead to a change in the activation distance, x^*, and consequently the activation volume with moisture content. To separate the various contributions to the moisture dependence of the activation volume will be difficult but to a first approximation we can assume that they vary in a linear fashion like the swelling behaviour of wood. Thus

$$v(W) = v(0)\,(1 + s_{\mathrm{vw}}) \tag{20}$$

where $v(0)$ is the activation volume of dry wood and s_{vw} is some volumetric swelling coefficient which we assume varies with moisture in the same way as the normal swelling coefficients of wood. Kollman and Côté [1] show that the swelling behaviour of wood is a linear function of moisture content up to almost the fibre saturation point and then remains essentially constant with increasing moisture content. Hence it follows that

$$v(W) = v(0)\left(1 + s_{\mathrm{v}}\frac{W}{W_{\mathrm{f}}}\right) \tag{21}$$

where s_{v} is the maximum volumetric swelling coefficient (i.e. s_{vw} at the fibre saturation point) and W_{f} is the moisture content at the fibre saturation point.

From the activation volume it should be possible to determine what fraction of the elementary fibril length takes place in the activation event. It is unlikely that the whole fibril is involved in the activation event for the energy required would probably be too high. It is more likely that something like a kink or a twist or a region of compression is formed on part of the fibril during the activation event, which then propagates down the fibril in the direction of the applied stress, thus giving fibril motion.

From Equations 6, 19 and 21 we obtain for the activation enthalpy the following expression

$$H(\tau^*, T, W) = H(T,0)\left(1 - a_{\mathrm{T}}\frac{W}{W_{\mathrm{f}}}\right) - v(0)\left(1 + s_{\mathrm{v}}\frac{W}{W_{\mathrm{f}}}\right)(\tau - \tau_{\mathrm{a}}). \tag{22}$$

Hence from Equation 14 we get

$$v = \left.\frac{\partial \Delta H(\tau^*, T, W)}{\partial \tau^*}\right|_{T,W} = v(0)\left(1 + s_{\mathrm{v}}\frac{W}{W_{\mathrm{f}}}\right) \tag{23}$$

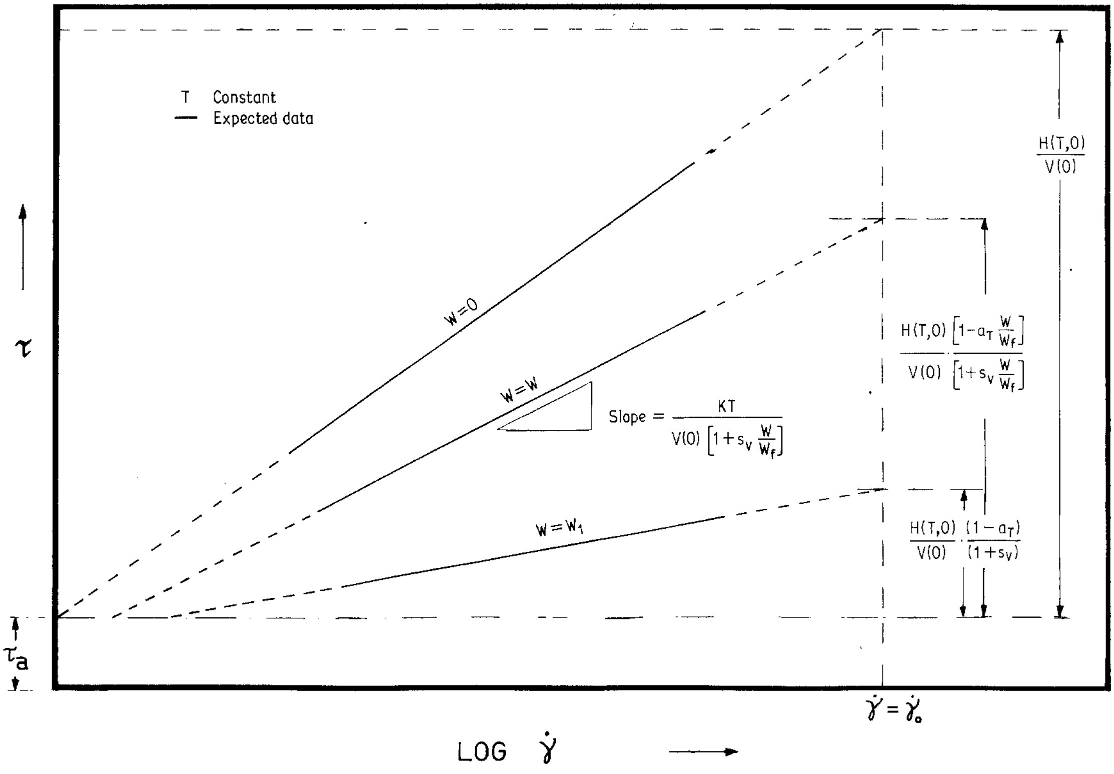

Figure 3 Schematic diagram of stress as a function of log strain-rate, for various moisture contents, at constant temperature.

and from Equations 7 and 22 an equation which describes the thermally activated behaviour of wood, namely

$$\tau = \tau_a + \frac{H(T,0)}{v(0)} \frac{\left(1 - a_T \frac{W}{W_f}\right)}{\left(1 + s_v \frac{W}{W_f}\right)} + \frac{KT \ln \dot{\gamma}/\dot{\gamma}_0}{v(0)\left(1 + s_v \frac{W}{W_f}\right)} \tag{24}$$

where W can have any value in the range 0 to W_f. The equation still holds when $W > W_f$ provided W/W_f is taken as equal to 1.

From Equation 24 it follows that when $W \geqslant W_f$

$$\tau(T, W \geqslant W_f) = \tau_a + \frac{H(T,0)}{v(0)} \frac{(1 - a_T)}{(1 + s_v)} + \frac{KT \ln \dot{\gamma}/\dot{\gamma}_0}{v(0)\,(1 + s_v)} \tag{25}$$

and hence $\tau(T,W \geqslant W_f) = \tau_f$, where τ_f is the strength of saturated wood. When $W = 0$, it follows that

$$\tau(T, 0) = \tau_a + \frac{H(T,0)}{v(0)} + \frac{KT \ln \dot{\gamma}/\dot{\gamma}_0}{v(0)} \tag{26}$$

where $\tau(T,0)$ is the strength of oven-dried wood.

We can examine schematically the behaviour exptected when stress is plotted against either W or log $\dot{\gamma}$. In Fig. 3 the variation of stress as a function of log $\dot{\gamma}$, for various moisture contents, at constant temperature is shown schematically.

3. Application

Yew and Ferguson [12, 13] have determined the longitudinal crushing stress of the softwoods *Pinus radiata* and *Kahikatea* as a function of strain-rate for various moisture contents, at room temperature. The data for *Pinus radiata* (see Fig. 4) follow very closely the schematic curves of stress versus log strain-rate shown in Fig. 3 in that for each moisture content the slope is constant and increases with decreasing moisture content, and the overall stress increases as the moisture content decreases. At strain-rates in excess of 10^2 sec^{-1} the curves start to rise steeply with increasing stain-rate. In this region some other mechanism is probably controlling the deformation behaviour. A similar trend has been observed by Perem [14] at high rates of loading. Apart from the behaviour at high strain-rates then, the data follow the general expectations of the theory. The data for *Kahikatea* (see Fig. 5) show the same general trends although the curves start to deviate from linear behaviour at a lower strain-rate, about 1 sec^{-1}. There is again the tendency for the stress to increase rapidly with increasing strain-rate for strain-rates greater than 10^2 sec^{-1}. The deviation from constant slope at a strain-rate of about 1 sec^{-1} may result from the onset of the mechanism controlling the behaviour of the highest strain-rates or it may result from a dependence of the activation volume on stress but this is unlikely for there is no such dependence at strain-rates less than 1 sec^{-1}. A decrease in $\dot{\gamma}_0$ with increasing strain-rate could account for the difference but a change of $\dot{\gamma}_0$ with strain-rate is usually attributed to the stress and hence the effect

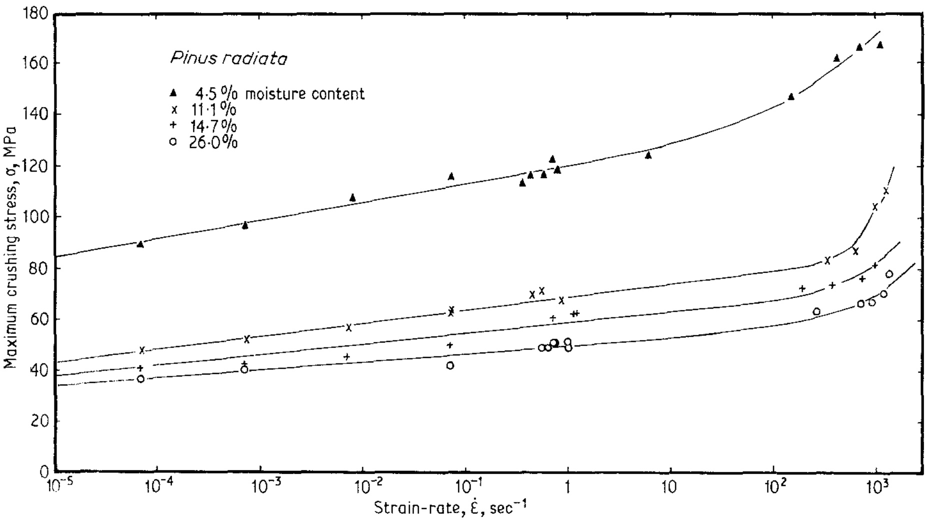

Figure 4 Stress–strain-rate curves in longitudinal compression.

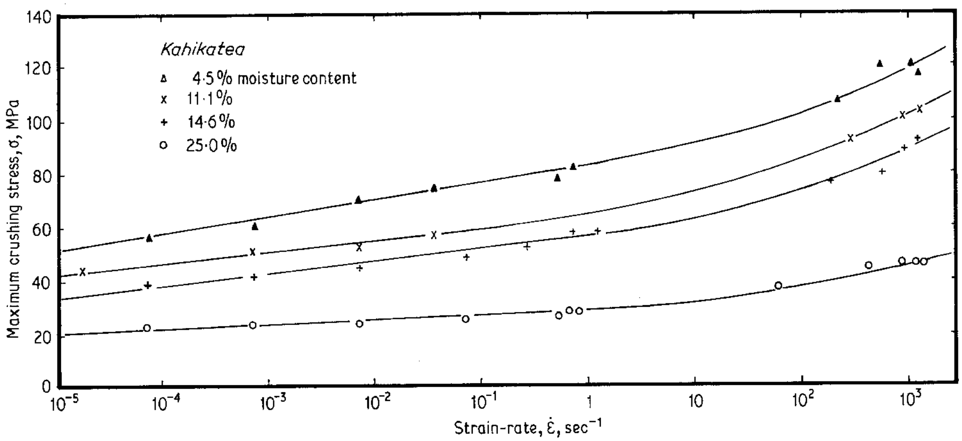

Figure 5 Stress–strain-rate curves in longitudinal compression.

is thus not likely to arise from this cause. As the effect does not appear to be related to stress or structure factor either the model proposed is inadequate or some other mechanism or mechanisms control the behaviour. At the highest moisture content investigated the strain-rate sensitivity of *Kahikatea* at strain-rates less than 1 sec^{-1} is very low indicating that the rate sensitivity of saturated wood would be even lower if not non-existent. This behaviour is indicative of there being a very small barrier energy (see Fig. 2) at 25% moisture which according to the model gets even smaller as the moisture content increases, so that when the wood is saturated the barrier height could be zero and the rate sensitivity also zero. In the present model this is equivalent to putting $a_T = 1$.

In Figs. 6 and 7 the apparent activation volume at various moisture contents is plotted as a function of stress for *Pinus radiata* and *Kahikatea*, respectively. It can be seen from these figures that over a wide range of stresses the apparent activation volume is independent of stress but a function of moisture content. At the end of the region of stress independence the activation volume drops rapidly with increasing stress. For *Pinus radiata* this corresponds to the region on the stress versus log (strain-rate) curve where the stress increases rapidly with increasing strain-rate and as mentioned before this probably results because some other mechanism is controlling the deformation. For *Kahikatea* the same behaviour is observed although in this case the rapid decrease in activation volume with increasing stress occurs at a lower strain-rate, about 1 sec^{-1}, on the stress

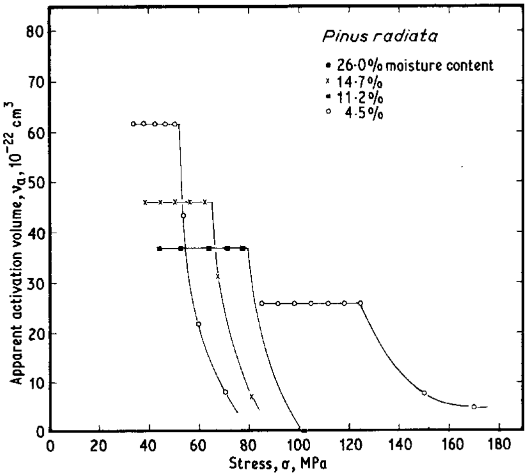

Figure 6 Activation volume of *Pinus radiata* versus stress.

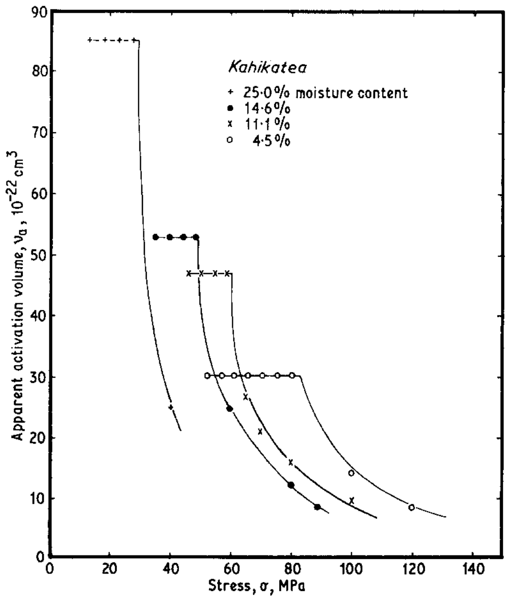

Figure 7 Activation volume of *Kahikatea* versus stress.

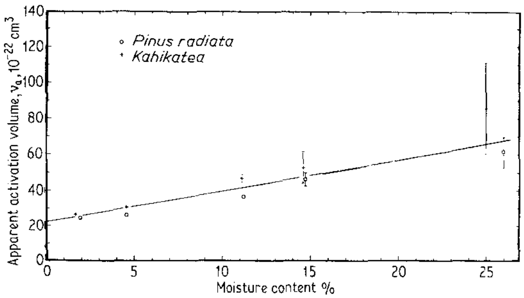

Figure 8 Activation volume versus moisture content.

versus log strain-rate curve. Again, as indicated above, this is probably indicative of a change in the mechanism controlling the deformation.

In Fig. 8 the apparent activation volume for the region of stress independence is plotted as a function of moisture content for both *Pinus radiata* and *Kahikatea.* The error bars indicate the increasing difficulty of determining the slope of the stress versus log strain-rate curves as the slope becomes small. The data would indicate that the activation volume is a linear function of moisture content in agreement with the assumptions of the theory and that for the two softwoods investigated the behaviour, within experimental accuracy, is not a function of wood species. The extrapolation of the data to zero moisture content gives an activation volume of $22.5 \times 10^{-22}\,\text{cm}^3$ which is approximately three times the volume of the cellulose molecule unit cell, $6.79 \times 10^{-22}\,\text{cm}^3$. The data would indicate that for the present results the activation volume varies from an equivalent of 4.4 cellulose unit cells at 4.5% moisture content to 9.7 at 25% moisture content. The equation of the straight line in Fig. 8 is given by

$$v_a = 1.73 \times 10^{-22}\,W + 22.5 \times 10^{-22} \tag{27}$$

where W is moisture content percent. If this equation is compared with Equation 22 and the moisture content at the fibre saturation point is taken as 0.28 [1] then s_v is 2.15 and Equation 21 becomes

$$v(W) = v(0)\left(1 + 2.15\,\frac{W}{W_f}\right) \tag{28}$$

where, for room temperature, 20° C, $v(0)$ has the value $22.5 \times 10^{-22}\,\text{cm}^3$. The value of 2.15 for s_v is much greater than that expected from the swelling behaviour of wood by a factor of about 20 times [1]. This just means that the activation volume increases more rapidly with moisture content than the overall volume but that both volumes still increase linearly with moisture content.

Recent papers by Mark [4] and Fengel 5] have shown that there is substantial evidence to suggest that elementary fibrils have diameters in the range 10 to 20 Å. If we take a fibril of diameter $D(0) = 20$ Å and consider the displacements during activation to occur in the longitudinal direction of the fibre, then at zero moisture content the activation volume will be given by

$$v(0) = \frac{\pi D^2(0)\,x(0)}{4}$$

and taking $v(0)$ as $22.5 \times 10^{-22}\,\text{cm}^3$ this gives the activation distance $x(0)$ as $\simeq 7$ Å. If we increase the moisture content the activation volume becomes to a first approximation

$$\begin{aligned} v(W) &= \frac{\pi D^2(W)\,x(W)}{4} \\ &= \frac{\pi D^2(0)}{4}\left(1 + \frac{2}{3}\,\alpha_{\text{vw}}\right) x(W) \end{aligned}$$

where α_{vw} is the normal volumetric swelling coefficient of wood. If we consider activation at 25% moisture content then $v(W) = 66 \times 10^{-22}\,\text{cm}^3$ and taking $\alpha_{\text{v},25}$ for Pine [1] as 0.09, then $x(25)$ the activation distance is about 20 Å. If we assume the activation event involves some sideways displacement of the fibril then the activation volume at $W = 0$ will be given by

$$v(0) = D(0)\,x(0)\,l(0)$$

where $l(0)$ is the length of fibril involved in the activation event. Taking the values at $W = 0$ as above then

$$x(0)\,l(0) \simeq 113 \times 10^{-16}\,\text{cm}^2.$$

If we assume that during the activation event the fibril is moved a distance $x(0)$ equal to its diameter $D(0)$ then $l(0) \simeq 5.6$ Å. Such a small activation length might imply that the fibril is sheared through. Peterlin [6] has suggested that the activation barrier energy for the sliding of fibrils may be as high, but not higher than that for bond rupture. Hence the above result is not inconsistent with the proposed theory. If we increase the moisture content and assume as above that the fibril is displaced a distance equal to its diameter then the activation volume becomes

$$v(W) = D(W)\,x(W)\,l(W)$$

$$= D(0)\left(1+\frac{\alpha_{vw}}{3}\right) x(0)\left(1+\frac{\alpha_{vw}}{3}\right) l(W)$$

$$v(W) \simeq D^2(0)\left(1+\frac{2\alpha_{vw}}{3}\right) l(W).$$

Taking the moisture content at 25% and the values of $v(W)$, $D(0)$ and α_{vw} as above then the length of fibril involved in the activation event at this moisture content, is $l(25) \simeq 16$ Å. If we consider the side ways displacement of the fibril to be in the form of a triangular shaped kink where the triangle is taken to be equilateral with height $x(0)$ equal to the activation distance and base $l(0)$ then at $W = 25\%$ moisture content, $x(25)$ and $l(25)$ are found to be 24 and 27 Å respectively. Dinwoodie [3] reports that the deformation of wood commences with kinking of the microfibrillar structure. Although the exact nature of the displacements during activation and the species being activated are not known with any degree of certainty the above simple calculations would indicate that the activation of the elementary fibrils is not inconsistent with the measured activation volume.

In the formulation of the theory we have assumed the fibril motion to be resisted by quasi-viscous forces and have taken these to be short-range barriers that can be overcome by stress assisted thermal activation. We have ignored, or at least assumed to be small, the viscous type interaction, i.e. one where $\tau \propto \dot{\gamma}$, and seemingly because of the relatively good qualitative fit of the model this was a reasonable thing to do. At high strain-rates where the fibrils will be moving fast the viscous forces might be rather large and we should then include a term in Equation 24 to take account of these. If we assume the viscous forces to be of the Newtonian type then we have

$$\tau = \tau_b + \eta(T,W)\dot{\gamma} \qquad (29)$$

where τ_b is given by the right-hand side of Equation 24 and $\eta(T,W)$ the viscosity coefficient is assumed to be small and a function of temperature and moisture content. Such a viscous force might explain the steep rise in stress with increasing strain-rate at high strain-rates in the stress versus log (strain-rate) curves.

4. Conclusions

The yield behaviour of wood is assumed to result from the stress-assisted thermally activated motion of elementary fibrils over short-range barriers. The barrier energy is taken to be a linear decreasing function of increasing moisture content and the activation volume a linear increasing function of increasing moisture content. The flow stress equation is given as

$$\tau(T,W) = \tau_a + \frac{H(T,0)}{v(0)}\,\frac{1-a_T\,(W/W_f)}{1+s_v\,(W/W_f)} + \frac{KT \ln \dot{\gamma}/\dot{\gamma}_0}{v(0)[1+s_v(W/W_f)]}.$$

The theory has been examined in relation to the strain-rate and moisture dependence of the crushing stress of *Pinus radiata* and *Kahikatea*. The data for these woods follow fairly closely the general expectations of the theory. The measured activation volume was found to be stress independent and to vary with moisture content in a linear manner. The magnitude of the activation volumes determined were not inconsistent with the thermally activated motion of elementary fibrils. The activation energy was not determined as all the tests were done at one temperature but it is hoped that the effects of temperature will be obtained in the near future.

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